Flow Patterns and Mixing Rates in Agitated Vessels

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Mixing rates in agitated vessels are predicted through measurement of the flow patterns which determine them. These measurements suggest the use of a model that assumes that nearly all the mixing occurs in a small "perfectly mixed" region near the impeller, with flow throughout the remainder of the tank serving primarily to bring the fluid into this region of the impeller.

On the basis of this model, equations were developed for relating volumetric flow rates, hence the mixing rates, to the operating variables. While the theory could be checked directly only to Reynolds numbers of slightly over 600 (owing to limitations of the experimental technique employed in this part of the mixing-rate studies), the volumetric flow rates could be measured from Reynolds numbers of 36 to 1.7 $\times$ 10⁴. The times required for completion of an acid-base neutralization (terminal mixing) were also measured from Reynolds numbers of 1.6 to 1.8 $\times$ 10⁵.

Flat-blade, dimensionally similar turbines with diameters of 2, 4, and 6 in. were used. Tank diameters ranged from 5.76 to 15.5 in. The baffle width equaled one tenth of the tank diameter in all runs. All the data were for Newtonian fluid systems, but the extension of this work to non-Newtonian materials is discussed briefly.

The literature in the field of agitation and mixing of fluids may be subdivided into two general categories: quantitative studies of power consumption and papers dealing with observations of the rates and quality of mixing. The former area has been investigated for a number of years, with both Newtonian and non-Newtonian fluids; however little has been done in regard to quantitative determinations of mixing rates or to an understanding of flow patterns and the factors controlling them. In fact, only two studies (*2, 12*) appear to be available in which any attempt was made to define the factors controlling mixing rates. In addition, several papers (*5, 6, 7, 10, 11*) report measurements of fluid velocities near the region of the impeller or in the vessel.

In view of these facts, the purposes of the present investigation were, 1) to study the flow patterns in agitated vessels and to determine how these factors influence mixing rates, and 2) to use the above to develop design equations for predicting mixing rates in Newtonian fluids and to consider the extension of these results to non-Newtonian systems.

The first of these purposes required the development of a suitable instrument for measuring point velocities under turbulent conditions in viscous liquids.

THEORY

Probably all the operations carried out in vessels equipped with an agitator depend primarily on high rates of fluid shear, high fluid-recirculation rates, or some combination of these variables. Emulsification processes, for example, require primarily a high intensity of the shearing action to reduce droplet size, and high rates of heat transfer depend upon large flow rates of the fluid past the heated surfaces. Chemical reactions require both large flow rates to distribute reactant streams throughout the vessel and a high intensity of turbulence (at least at one place within the vessel) to aid the mixing of the reactant streams to the desired degree of completeness on a molecular level.

Metzner and Taylor (*5*) have shown that the shear rates and local rates of power dissipation in a fluid agitated inside a cylindrical vessel with flat-bladed turbines are initially very high at the impeller but decrease by several orders of magnitude within about 1 in. from the tip of a 4-in. turbine impeller. Obviously most of the power is dissipated in the immediate region of the impeller. The fluid in this region has a more thoroughly mixed condition than anywhere else in the tank, regardless of the measure or standard of quality of mixing. Under these conditions the mixing process may be described by a gross model which assumes a "perfectly mixed"* volume in the region of the impeller with little mixing in the rest of the tank. For clarity of expression, the equations for this model will be formulated for a specific mixing operation, the batchwise mixing of the reacting components of an infinitely fast chemical reaction. The latter restriction is introduced to eliminate any dependence on chemical kinetics (the sole controlling factor will be the rate of mixing of reactants) and should be a good assumption for most ionic reactions. The resulting equations may also be used as an approximation to the residence times required in continuous stirred-tank reactors, but these introduce the additional complication that some reactants may short-circuit to the draw-off point, especially if it is poorly located, and additional information as to the distribution of residence times would then be required.

The neutralization of a given number of equivalents of base (M_o) initially evenly distributed throughout a tank of volume V_T by the stoichiometric amount of acid introduced near the impeller* at time zero will be considered. The following additional assumptions are utilized:

1. Nearly all the mixing takes place in a small perfectly mixed volume of fluid near the impeller.
2. Any unreacted acid which is pumped out of this volume reacts in the other parts of the tank in localized pockets of turbulence before being recirculated to the impeller. While the intensity of such turbulence away from the impeller is low, that it is not zero at Reynolds numbers above 10 has been shown both by photographic data (*5*) and by visual observations in the present study. In particular the regions behind the baffles, above and below the impeller (near the vertical axis of the tank), and near the tank wall at the level of the impeller show appreciable turbulence at transitional and turbulent Reynolds numbers.
3. Any base which enters V is neutralized substantially instantaneously.
4. All the fluid in the tank is in recirculating motion.

Under these conditions a material balance for the unreacted acid, written over the volume, simplifies to

$$-QC_t - QC_T = V\, dC_t/dt \qquad (1)$$

The two limiting relationships between time and C_T are as follows. If the outflow from V short-circuited completely, then shortly after time zero

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* Perfectly blended, but not necessarily homogeneous on a molecular scale.

* In continuous processes all reactants would normally be introduced into this perfectly mixed region.

C_T would equal zero and neutralization of the base would occur only by diffusion. On the other hand, if the outflow from V did not recirculate to V until all the fluid in the tank had passed through it once, then C_T could be taken as equal to M_o/V_T. In reality the value of C_T always lies somewhere between these limits, and because base is consumed in the reaction, C_T is probably a decreasing function of time. Additionally C_t and C_T have the following similar characteristics:

$$C_T(0) = M_o/V_T \qquad (2a)$$

$$C_t(0) = (M_o/V)(1 - V/V_T) \qquad (2b)$$

$$C_T(t_F) = 0 = C_t(t_F) \qquad (2c)$$

These considerations suggest that a good approximation for C_T as a function of time would appear to be

$$C_T = (V/V_T)C_t \qquad (2d)$$

which is equivalent to assuming that the concentration of base in the inflow stream to the impeller is equal to the average base concentration in the tank.

Substituting Equation ($2d$) in Equation (1) and integrating, one obtains

$$C_t/C_t(0)2 = e^{-(Q/V)(1+V/V_T)t} \qquad (3)$$

or

$$M(t)/M_o = (1 - V/V_T)\, e^{-(Q/V)(1+V/V_T)t} \qquad (4)$$

If one ignores the short time lag between the pumping of a slug of acid out of the volume and its neutralization in the tank, $M(t)/M_o$ is equal to the fraction of the original acid or base still unreacted at t; therefore

$$\frac{1 - F(t)}{1 - V/V_T} = e^{-Q/V(1+V/V_T)t} \qquad (5)$$

The following approximate conclusions concerning the volume may be drawn from photographic data (5):

1. V is approximately independent of impeller speed.

2. $0 < V \leqq 0.15$ cu. ft. for the impeller diameter range from 2 to 6 in.

The volumetric flow rate is not only of importance in Equation (5) and the problem considered here but also in almost all other mixing-rate processes. The following expression for the theoretical pumping capacity of a flat-blade turbine impeller has been derived by van de Vusse (*12*):

$$Q_{th} = \pi^2 ND^2w\,(1 - q^2)^{1/2} \qquad (6)$$

This derivation, based on the analogy between a turbine mixer and a centrifugal pump, assumed that loss of energy due to drag on the impeller is negligible. Entrainment of fluid by the jet stream from the impeller was also neglected. It appears reasonable to rewrite Equation (6) as

$$Q = c_1\pi^2 ND^2w(1 - q^2)^{1/2}f(\mu) \qquad (7)$$

Fragmentary data (*10*) indicate, that c_1 is independent of impeller speed and is equal to 1.96 for the agitation of water by a 4-in. turbine impeller operating in the impeller speed range of 100 to 200 rev./min. in a baffled tank.

Equation (7) is a somewhat discouraging result. The factor c_1, and possibly $f(\mu)$, appear to be important enough to require careful determination. At present this must be carried out experimentally. Similar conclusions may be reached from the more recent and extensive analysis of Nielsen (7). Thus for the present the work required to obtain Q [hence the mixing rates obtained by use of Equation (5)] appears to be nearly as extensive as a direct (empirical) determination and correlation of mixing rates. This is not to say that the previous theoretical considerations are of no value, but they do not afford any very immediate prediction of mixing rates.

An empirical analysis of the terminal mixing time yields by means of dimensional considerations

$$f(ND^2\rho/\mu,\ T/D,\ y/D,\ g/N^2D,\ Nt_F = 0 \qquad (8)$$

The Froude number g/N^2D accounts for the gravitational effects on the fluid motion. In view of the fact that several investigators have shown that the Froude number is not of significance in determining power requirements in baffled tanks, one might conclude superficially that the Froude number should not affect mixing times under similar conditions. However the effect of gravity on the flow pattern, hence on mixing rates, can be shown most easily by considering the limiting case of zero gravitational force. Under these conditions the liquid would be thrown out to the sides of the vessel by the impeller and would tend to remain there, yielding no recirculation and therefore little or no mixing. Thus if the force of gravity could be changed in the laboratory, the flow pattern and therefore the mixing time would be affected unless the vessel were filled to a rigid surface.

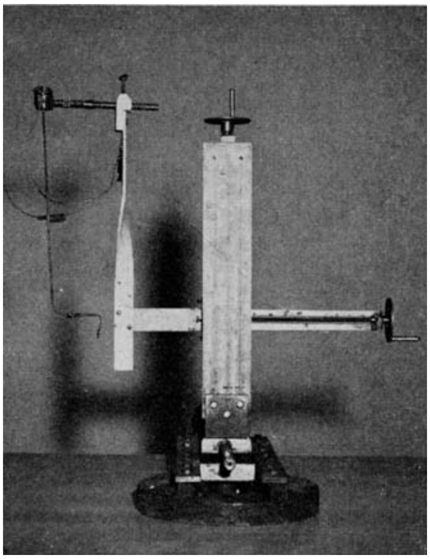

Fig. 1. Thermistor probe and probe positioner.

DESCRIPTION OF EQUIPMENT

The data were taken in baffled (J/T = 0.1), cylindrical, flat-bottomed, Pyrex-glass mixing vessels. The fluids were agitated with centrally positioned turbine impellers having six flat blades (w = $D/5$). Each of the various-sized impellers was mounted 35.4% of the distance from the tank bottom to the liquid surface. The impellers were geometrically similar in all dimensions. The size ranges of impeller diameter and vessel diameter which were used in the study are given in Table 1.

The probe used to measure flow rates and the probe positioner are shown in Figure 1. The probe was formed by joining a long piece of L-shaped hypodermic tubing to a short L-shaped piece by a collar-and-spring device in such a way that the short L could be rotated independently of the large L. A thermistor bead was mounted at the tip of the short L, and the leads were threaded through the tubing. The probe positioner consisted of three lathe beds arranged so that the probe might be moved independently in each of three directions, and the sides of the beds were marked at intervals to permit determination of the exact position of the probe in the tank.

The velocity-measuring probe was used as the fourth side of a Wheatstone's bridge; two 500-ohm resistors and a decade resistor comprised the other three sides. The bridge was powered by a power supply and a power amplifier; input and output voltages were measured with suitable voltmeters. The resistance of the probe was allowed to vary with flow rate, and the input to and the output from the bridge were kept constant by varying the decade box resistance. This method of operation gave a calibration curve of velocity as a function of the decade box reading and the viscosity and density of the fluid.

The test liquids, mixtures of corn syrup and water, were all single phase Newtonian liquids. Their viscosities were measured with a capillary-tube viscometer.

OPERATING PROCEDURE

Mixing times were measured by means of a slightly modified version of the method of Fox and Gex (2). The time necessary to neutralize a known amount of acid dispersed in a tank of fluid with an exact equivalent amount of base is defined as the terminal mixing time (2). Fractional mixing times were similarly measured by noting the time for some known fraction of the stoichiometric amount of acid to be neutralized. Methyl-

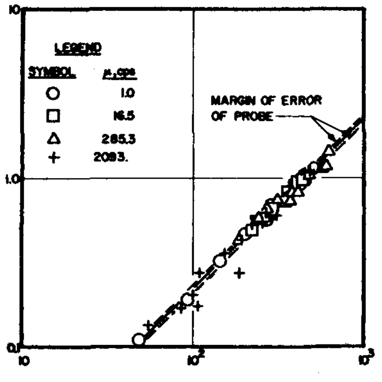

Fig. 2. Calibration of Probe, with photographic data used as the primary standard.

Table 1. Ranges of Variables Studied

Variable	Range: Mixing-time data	Range: Velocity data
Impeller diameter	2 to 6 in.	2 to 6 in.
Tank diameter	5.72 to 15.5 in.	11.3 in.
T/D ratio	1.35 to 5.66	1.89 to 5.66
Fluid depth	6 to 12 in.	11.3 in.
y/D ratio	1.5 to 6.0	2.8
Impeller speed	40 to 4,820 rev./min.	75 to 2,300 rev./min.
Reynolds number	1.6 to 1.83 $\times 10^5$	35.7 to 1.72 $\times 10^4$
Viscosity	0.3624 to 7,780 centipoises	1.0 to 1,783 centipoises

All data were obtained with $J = 0.1T$.

Density terms have been included in Figures 3 and 5 since they generally accompanied changes in viscosity; however the changes in fluid density were too small to enable a highly accurate determination of the true effect of density changes alone.

red indicator was used to determine the end points. At the initial neutral point the liquid had a pH such that the indicator was just on the yellow side of the red-yellow color change (pH = 6.3). Since methods of measurement involve a chemical reaction between ions, a degree of mixing close to absolutely complete mixing (homogeneity even to the molecular level) must be achieved before all the red color can be eliminated, when it is assumed that the visual observation is sufficiently acute to detect any unmixed volumes.

The thermistor probe was calibrated by means of velocity data obtained by several investigators (*5, 10, 11*) in photographic studies of the flow patterns in the region of a turbine impeller. These data were taken with mixing equipment identical to that which was used in a part of this work. The data were obtained by the use of corn-syrup–water mixtures of four viscosities: 1.0 centipoises (*10*), 16.5 and 285.3 centipoises (*11*), and 2,093 centipoises (*5*). The final calibration curve, Figure 2, shows that most of the data are within the margin of error of the probe, as determined by the sensitivity of the vacuum-tube voltmeter to decade resistance changes. The detailed function $F(\eta, D'_o)$ is not given, as it is undoubtedly restricted to the particular probe used. Figure 2 is given primarily to show that point velocities could be determined with a mean accuracy of ±5% over the entire range studied. A detailed description of the probe and its directionality and operation is available (*8*).

Total flow rates were obtained by integrating the measured velocities over the entire moving stream of fluid, from the center of the vertical vortex above the impeller to the center of the vortex below. These vortices are very well defined under conditions of good mixing (*5, 10*) and delineate completely and accurately the area over which such an integration should be carried out.

RESULTS AND DISCUSSION

The study of mixing rates and flow patterns covered the major areas of industrial importance with regard to Reynolds number and geometric ratios; however the scale of equipment was smaller than would be found in industry. The 498 mixing-time measurements and twenty-five velocity traverses were taken over the ranges of variables given in Table 1.

Volumetric Flow Rate

The volumetric flow-rate data (Figure 3) were correlated by

$$Q = 9.0 \times 10^{-4} ND^2 w (D^{0.4}\rho/\mu)^{0.5} (1 - q^2)^{0.5} \quad (9)$$

over the entire Reynolds number range studied. The maximum deviation from the best line through the data is 33%; the average deviation of the twenty-five points is 9.7%.

Calculation of the volumetric output of an impeller for design purposes, with either Figure 3 or Equation (9) used, requires some knowledge of the term $(1 - q^2)^{1/2}$. In the present work q was measured; it almost always proved to be negligible, as suggested by van de Vusse (*12*) for baffled systems. In one instance omission of this term would have resulted in an error of 7%, but in no other case was the term less than 0.99. Thus the present recommendation is to neglect it completely.

Comparison of Equations (9) and (7) shows that the product $c_1 f(\mu)$, while independent of impeller speed

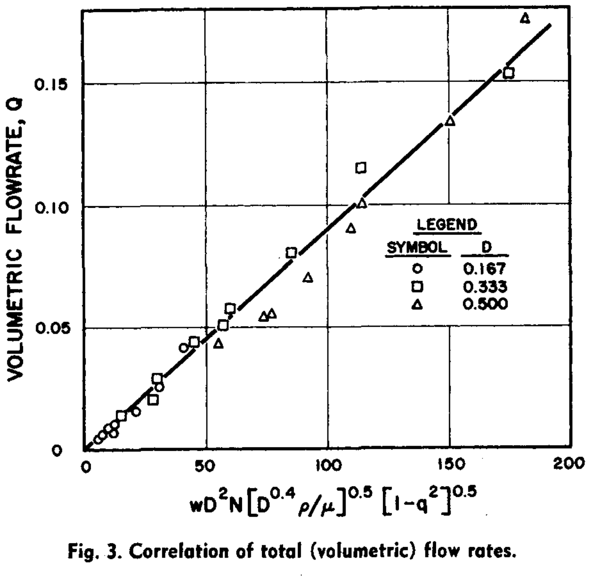

Fig. 3. Correlation of total (volumetric) flow rates.

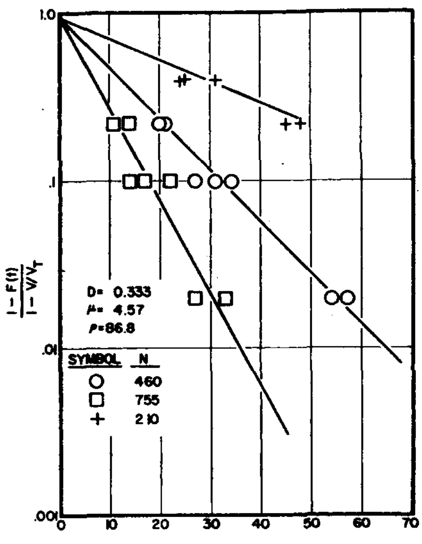

Fig. 4. Proof of model used to correlate times required for fractional completion of the mixing.

TABLE 2. SCALING EQUATIONS—TURBULENT REGION

Reference	Operating conditions	Mixing time equation	Slope of logarithmic plot of N_P vs N_{Re}	Scaling equation for geometric similarity and equal mixing times
1	General	—	—	$P_2 = K^5 P_1$
2	Propeller, unbaffled	$\frac{t_F (ND^2)^{2/3} g^{1/6}}{y^{1/2} T} \alpha \left(\frac{D^2 N \rho}{\mu}\right)^{-1/6}$	—1/6	$P_2 = K^{4.1} P_1$
3	Baffled, propellers	$t_F \alpha N^{-1}$	0	$P_2 = K^5 P_1$
12	Paddles or turbines, unbaffled	$t_F \alpha N^{-1}(N^2 D)^{-0.30 \text{ to } 0.35}$	0 to —0.15	$P_2 = K^{4.11} P_1$ to $K^{4.44} P_1$
12	Propellers, baffled and unbaffled	$t_F \alpha N^{-1.5} D^{-0.25}$	0	$P_2 = K^{4.50} P_1$
Present work	Turbines, baffled	$\frac{t_F (ND^2)^{2/3} g^{1/6}}{y^{1/2} T} = \text{const.}$	0	$P_2 = K^{5.75} P_1$

(as postulated earlier), is incorporated as a dimensional term which differs appreciably from a value of unity. Therefore extrapolation to greatly different values of the relevant variables cannot be recommended. This limitation is unimportant insofar as Reynolds numbers and viscosities are concerned, since wide ranges were covered, but the applicability of Equation (9) to large-scale equipment remains unproved. An article appearing since the present work was completed (*6*), while very carefully carried out, does not alleviate this problem, nor does it propose any general correlation of the recirculation rate Q.

Fractional Mixing Times

The times required to neutralize between 60 and 98% of the base in the tank, by addition of the appropriate percentage of the stoichiometric amount of acid, are shown in Figure 4 for three typical runs. The coordinates and scales are as suggested by Equation (5); the fact that the correlating curves are straight lines shows that Equation (5) is obeyed. Knowledge of the value of Q from the flow-rate measurements enables determination of V from these slopes*. Values of the perfectly mixed volume as obtained from twenty-five sets of runs similar to those shown on Figure 4 are correlated in Figure 5. It is evident that this volume is determined primarily by the impeller size and is only slightly dependent on the fluid properties; an 1,800-fold change in viscosity changes the value of V by only about 40%. No dependence on impeller speed was shown over the entire range studied, 75 to 2,300 rev./min.

Figure 6, with the above correlations for V and Q utilized, summarizes the applicability of Equation (5) to the prediction of the actual fractional mixing times. The maximum deviation from the theoretical line is 26.5%, and the average deviation is 10.3%; the range of Reynolds numbers covered is from 36 to 640. Higher Reynolds numbers could not be studied, because the time for fractional mixing became too short. At low Reynolds numbers not all the fluid in the tank is in motion, the theory becomes inapplicable as mixing rates approach zero, and the process is of no industrial interest. It must also be noted that the theory predicts infinite mixing times for complete or perfect mixing [$F(t) = 1.0$]. While the theory must break down as $F(t)$ approaches unity, since the diffusive processes largely neglected in the present analysis must be responsible for the ultimate completion of mixing, no deviations were noted up to 98.8% of complete mixing, the highest level studied in this phase of the program.

* Since $V/V_T << 1.0$ only a very approximate estimate suffices to define the value of the ordinate for any given measurement; the slope however is sensitive to small percentage variations in V.

Time Required for Terminal Mixing

The relationship between the dimensionless groups in Equation (8), for calculation of the time required for terminal mixing, is shown in Figure 7. The average deviation of all the points from the best curve through the data is 11.4%. The maximum deviation is

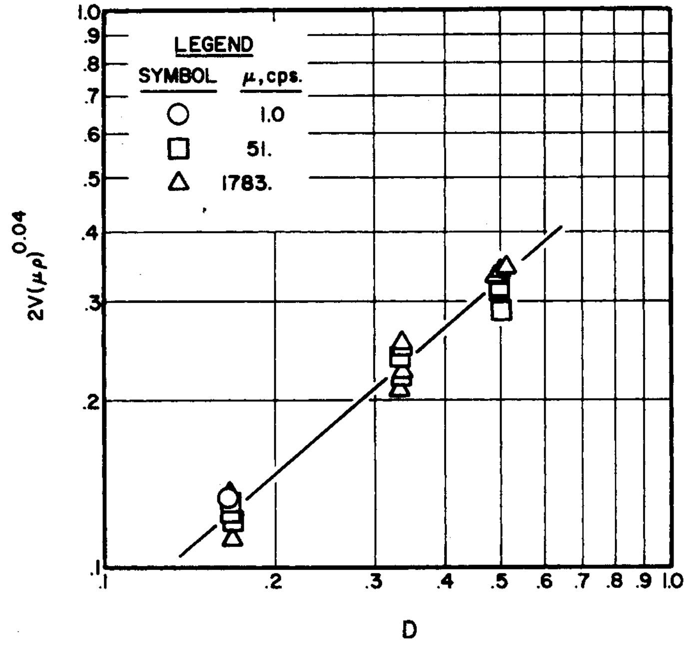

Fig. 5. Dependence of perfectly mixed volume on impeller diameter, fluid density, and viscosity.

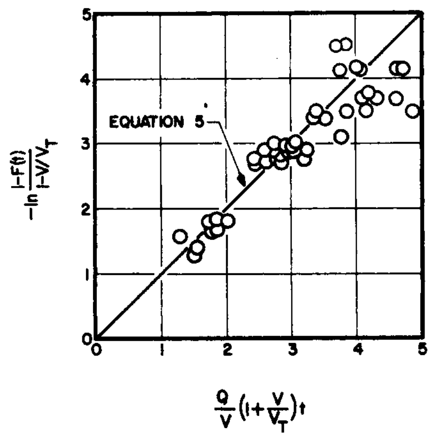

Fig. 6. Final correlation of fractional mixing times (V and Q obtained from Figures 5 and 3, respectively).

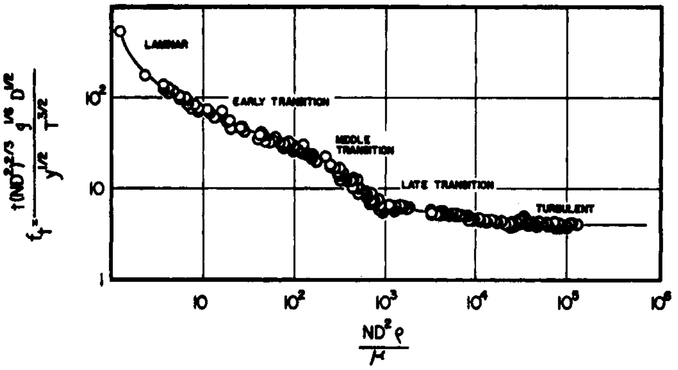

Fig. 7. Empirical correlation of time required for terminal mixing.

about 40%, but most points are within 20% of the curve.

The end of the laminar region, in which little or no turbulent mixing takes place, coincides approximately with the lowest Reynolds number at which flow at the tank wall and fluid surface may be first observed visually (*4*, *5*). Photographs taken (*5*) at Reynolds numbers in the early transition region show that all the fluid in the tank is in motion, but the flow is very sluggish, largely in the tangential direction, and the mixing rates are still very low. No turbulence is evident except close to the impeller. Photographs in the middle transition region show much more turbulence, and the velocity measurements show that all the fluid is now in reasonably rapid motion; therefore the mixing times decrease rapidly with further increases in flow rate or Reynolds number.

Although mixing times, even for terminal mixing, were too short to be measured above a Reynolds number of 1.8×10^5, the turbulent region of the curve was extended after a comparison of the existing mixing-time–factor curve with the power-number–Reynolds-number curve for turbine impellers rotating in baffled tanks, in both Newtonian (*9*) and non-Newtonian (*4*) fluids. Figure 8 shows both smoothed curves. The important points to note are as follows:

1. As one proceeds up the Reynolds-number scale, the mixing-time–factor curve changes slope each time and at the same point that the power curve changes. This is only reasonable in that a change in the trend of power requirements indicates that the flow patterns in the tank must be changing, which in turn lead to changes in mixing rates.

2. The power curve flattens out, that is the power requirements become independent of viscosity, at a Reynolds number of about 2 to 4×10^4 and remains flat to a Reynolds number of 10^6, the upper limit of the power data.

In view of these facts, moderate extensions of the mixing-time–factor curve seem to be reasonable.

Comparison with Prior Art

Fox and Gex (*2*) have developed a mixing-time-factor–Reynolds number curve for Newtonian fluids agitated by marine propellers and side-entering jets in unbaffled tanks. Several differences between their work and the present study are of importance:

1. Their final correlating equation is identical in form to the present one for turbine impellers with two exceptions: the mixing-time factor used for turbines is equal to the product of the mixing-time factor for marine propellers or jets and the square root of the D/T ratio, and second the numerical values are generally different for different impellers even after the D/T term is considered.

2. The disappearance of the last wisp of color in their experiments was isolated and located at different points in the tank, whereas the disappearance of the last wisp of color with turbine impellers always occurred in the region of the impeller.

These differences between turbine mixers on one hand and jets and propellers on the other indicate that a significant portion of the mixing in the latter two systems occurs away from the region of the agitator mechanism. At very high Reynolds numbers this factor, rather than the rate of fluid recirculation, may possibly control mixing rates, and the propeller becomes a more efficient mixer than does the turbine. Within the range of Reynolds numbers studied, however, the reverse appears to be true. For example, with a 2,000-centipoise fluid, a 3-in. impeller, and a T/D ratio of 6.0 in both systems the power consumptions are equal when the Reynolds numbers are 100 for the propeller and 60 for the turbine. The mixing time with a turbine used under these conditions is only about 6% of that required by the propeller. If a fluid having a 1-centipoise viscosity were used instead, the mixing times would be identical in both systems at equal power consumptions. In this case the Reynolds numbers are 2×10^5 for the propeller and 0.7×10^5 for the turbine. Kramers and co-workers (*3*) arrived at the opposite conclusion concerning the relative merits of propellers and turbines, but their turbines had unusually narrow blades, to which their inferior performance may have been due.

Corrsin (*1*) presented an analysis of a fluid mixer operating at high Reynolds numbers (turbulent region) in which he assumed that stationary isotropic turbulence exists throughout the tank. From this analysis he derived an equation for scaling geometrically similar mixers. This and the other scaling equations available at the present time are presented in Table 2. One may note that although all of the exponents for the scaling factor K are between 4 and 6, only the data of Kramers *et al.* (*3*), which were used by Corrsin to help support his deduction, yield an equation identical with the postulated one. However Kramers's results are not entirely conclusive in that his group carried out only a small number of tests in which the scale of the equipment was varied. While Corrsin's approach is thus left with little direct

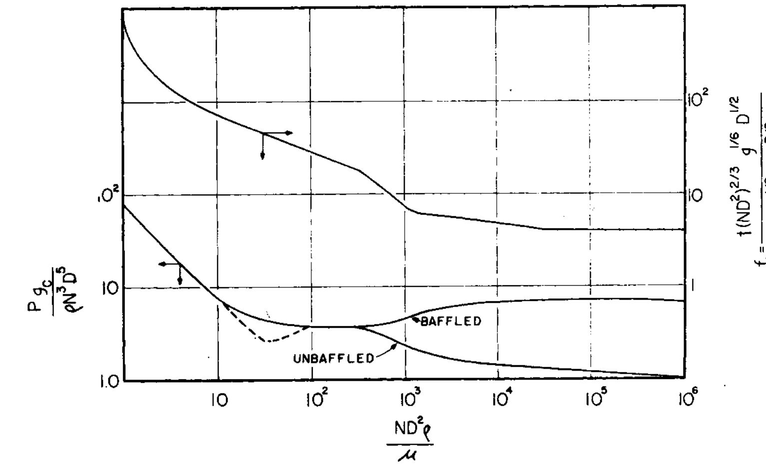

Fig. 8. Comparison of terminal mixing time and power consumption correlations.

experimental support, it should be noted that the empirically determined exponents are subject to considerable experimental error and, with the exception of the present results (on small-scale equipment), have not been determined to better than 10 to 20%.

Mixing Rates in Non-Newtonian Systems

Photographic (5) and visual (4) data on pseudoplastic non-Newtonian fluids agitated with a single flat-bladed turbine show that only a part of the fluid in the tank is in motion unless the Reynolds numbers exceed about 270 at $T/D = 3.0$, 110 at $T/D = 2.0$, 90 at $T/D = 1.5$, and 50 at $T/D = 1.3$

Therefore the mixing-time factor of Figure 7 would tend to approach very large values below these Reynolds numbers, as compared with the break at a Reynolds number of about 2 for the Newtonian data in Figure 7. In other words a series of laminar curves, each intersecting the early-transition curve of Figure 7 at the above-indicated Reynolds numbers, for various T/D ratios, might be expected. Since the photographic study (5) indicated rapid increases in fluid turnover at increasing Reynolds numbers, use of the curve of Figure 7 at higher Reynolds numbers would perhaps be conservative for pseudoplastic fluids.

Use of the curves of Figure 7 for dilatant systems is also suggested in the absence of specific data, but no estimate of the accuracy of this suggestion is possible.

CONCLUSIONS

1. A model of the mixing process, based on the assumption that the contents of the vessel consist of a small perfectly mixed volume in the region of the impeller, surrounded by unmixed fluids, was supported experimentally to Reynolds numbers of 640. This model may be used to define the extent of completion of fast, batch reactions by the equation

$$\frac{1 - F(t)}{1 - V/V_T} = e^{-Q/V(1+V/V_T)t}$$

The model probably will not apply to mixing in the fully turbulent region, since considerable mixing may then occur in other parts of the tank and the volume may cease to be a pertinent variable. All data supporting this model were taken with a flat-blade turbine in a baffled tank.

2. Data on the total volumetric flow rates were correlated over the Reynolds number range of 36 to 1.72×10^4 by

$$Q = \frac{9.0 \times 10^{-4} NwD^2 (D^{0.4}\rho/\mu)^{1/2}}{\sqrt{1-q^2}}$$

3. Times required for terminal mixing, measured by carrying out an acid-base neutralization, were correlated empirically (Figure 7) for Reynolds numbers between 3 and 1.8×10^5. Each change in the character of the mixing-time curve reflects a change in the flow conditions in the tank which is also reflected in the power-number–Reynolds-number curve.

4. The mixing-time factor for the agitation of Newtonian fluids by turbine impellers in baffled tanks is found to be related to the mixing-time factor for the agitation of Newtonian fluids by jets and propellers in unbaffled tanks by the square root of the T/D ratio. The mixing-time factors and power-consumption data combined suggest that turbines are much more efficient at low Reynolds numbers than are propellers. At the highest Reynolds numbers studied in this work the two have become comparable.

5. Application of the present mixing-time data (Figure 7) to non-Newtonian systems at sufficiently high Reynolds numbers is suggested as an approximate procedure for pseudoplastic systems until data become available.

ACKNOWLEDGMENT

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NOTATION

c_1 = entrainment factor
C_T = concentration of base, moles/cu. ft. $C_T(0)$ refers to this term at zero time
C_t = concentration of acid, moles/cu. ft. $C_t(0)$ refers to this term at zero time
D = impeller diameter, ft.
D_o = probe resistance at zero flow, ohms
D_R = probe resistance at velocity u, ohms
D'_o = $(D_R - D_o)$, ohms
e = base of Napierian logarithms
f = functional relationship
f_t = mixing-time factor (dimensionless) $f_t = t_f\,(ND^2)^{2/3}\,g^{1/6}\,D^{1/2}/y^{1/2}T^{3/2}$
$f\eta, D'_o$ = abscissa of calibration curve
$f(\mu)$ = correction factor for viscous drag of fluid on the impeller
$\overline{F(t)}$ = fraction mixed, dimensionless
g = gravitational acceleration, ft./sec.^{2}
g_c = conversion factor, ft. lb.-mass/(lb.-force) (sec.)^2
J = baffle width, ft.
K = ratio of size, prototype to model
$M(t)$ = moles of acid or base in tank at time t
M_o = moles of acid or base in tank at time zero
N = rotational speed, rev./sec. in dimensionless groups; otherwise rev./min.
N_{Re} = Reynolds number (dimensionless), $ND^2\rho/\mu$
N_P = power number (dimensionless), $Pg_c/N^3D^5\rho$
P = power, (ft.) (lb.-force)/sec. P_2 refers to the power consumption of a prototype and P_1 to that of a model.
Q = volumetric flow rate, cu. ft./sec.
Q_{th} = theoretical volumetric flow rate, cu. ft./sec.
q = ratio of (angular) fluid velocity at the blade tip to angular impeller velocity
T = tank diameter, ft.
t = time, sec.
t_F = terminal mixing time, sec.
u = velocity, ft./sec.
V = volume of perfectly mixed region, cu. ft.
V_T = total volume of fluid in tank, cu. ft.
w = impeller blade width, ft.
y = fluid depth, ft.
η = ratio of (μ/ρ) for unspecified fluid to (μ/ρ) for water, dimensionless
μ = viscosity, (lb.-mole) (ft.) (sec.)
π = 3.14 . . .
ρ = density, lb.-mass cu. ft.

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